Math 53: Multivariable Calculus **Sections 212, 213**

Worksheet for 2020-09-09

Problem 1. The two space curves $\mathbf{r}_1(t) = \langle 2t, 2-2t, 3+t^2 \rangle$ and $\mathbf{r}_2(t) = \langle 6-2t, 2t-4, t^2 \rangle$ intersect. Find the coordinates of the point of intersection, and find the angle formed by the two curves at that point of intersection.

To find intersection:

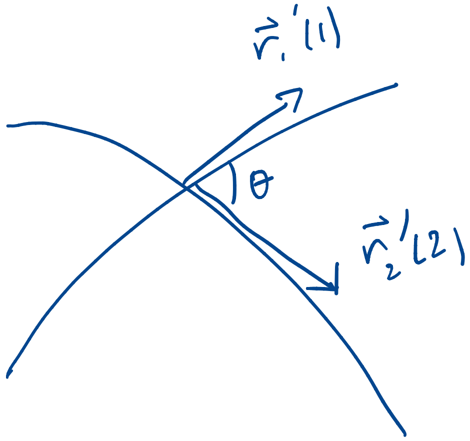
$$\begin{cases} 2t = 6-2s \\ 2-2t = 2s-4 \\ 3+t^2 = s^2 \end{cases}$$

this is redundant actually

$t = 3-s$, substitute into 3rd eq: $3+9-6s+\cancel{s^2} = \cancel{s^2}$

$s = 2$ and $t = 1$.

Indeed: $\vec{r}_1(1) = \langle 2, 0, 4 \rangle = \vec{r}_2(2)$.



Now use $\cos\theta = \dfrac{\vec{r}_1'(1) \cdot \vec{r}_2'(2)}{|\vec{r}_1'(1)|\,|\vec{r}_2'(2)|}$

$\vec{r}_1'(1) = \langle 2, -2, 2 \rangle$

$\vec{r}_2'(2) = \langle -2, 2, 4 \rangle$

$\cos\theta = 0$ so $\theta = \boxed{\pi/2}$.

(Note: if we found an obtuse angle, would be better to give $\pi - \theta$ as answer)

Problem 2. Find a function $f(x, y)$ such that, for every nonnegative number k, the level set $f(x, y) = k$ is a circle of radius $2k$ centered at the point $(2, 3)$.

What kind of shape is the surface $z = f(x, y)$?

circle of radius $2k$ centered at $(2,3)$:

$$\frac{(x-2)^2}{4k^2} + \frac{(y-3)^2}{4k^2} = 1$$

$$\frac{1}{2}\sqrt{(x-2)^2 + (y-3)^2} = k$$

So $f(x,y) = \frac{1}{2}\sqrt{(x-2)^2 + (y-3)^2}$.

This is (the top half of) a cone.